Math

Level	Summary	Description
Level 1	**Quantity Comparison, Subitizing & Attribute Sorting (Shapes + Logic Foundations)**	Children develop foundational number sense through joyful, hands-on exploration in a prepared environment. They instantly recognize small quantities (subitizing 1–4 objects or dots) without counting one-by-one. They compare sets using precise language: 'more than,' 'fewer than,' or 'the same number/equal,' and justify by physically matching objects or counting. Simultaneously, they explore everyday shapes (circles, squares, triangles, rectangles) in the classroom, playground, and home—naming them, sorting by attributes (straight vs. curved sides, number of corners/vertices), and classifying objects by color, size, or shape. This builds early geometric awareness and logical thinking (same/different, rule-based grouping). They connect spoken number words to quantities and numerals 0–5. Simple AB patterns appear through movement, sound, and objects. Real-life links: counting snacks or friends, sorting toys/laundry, shape hunts. Teachers model thinking aloud and use games like 'Quick Look' dot cards and attribute blocks. This level honors the sensitive period for order and exactness while weaving in geometry and logic from day one—preventing the 'counting-only' bottleneck of traditional approaches.
Level 2	**Counting to 10 with One-to-One Correspondence, Numeral Connections & Repeating Patterns**	Children count reliably to 10 (and beyond in play) using one-to-one correspondence, understanding that the last number spoken tells 'how many' in the whole set. They compare small groups and explain why one has more or less using matching or counting strategies. Numeral recognition, reading, and writing 0–10 is solidified through meaningful matching games and dictation. Part-whole thinking emerges as children compose and decompose numbers to 10 with concrete materials (e.g., '5 is 2 and 3'). Repeating patterns (AB, AAB) are extended using objects, shapes, movements, and sounds—building algebraic thinking foundations. Geometry continues: children sort shapes by multiple attributes and begin simple pattern-block designs. Real-life applications include counting steps, setting tables, or organizing classroom materials. Teachers emphasize 'counting on' from known quantities and model explaining strategies. This level balances deeper counting fluency with continued geometry, patterns (early algebra), and logic while avoiding overload by integrating rather than isolating skills.
Level 3	**Part-Part-Whole Thinking, Shape Properties, Time Sequencing & Money Play**	Children deepen understanding that numbers are composed of parts that make wholes (part-part-whole for 5–10 and beginning teens). They recognize, read, write, and match numerals 0–10 fluently while beginning to see teens as '10 and some more.' Geometry expands: children explore properties of 2D shapes (sides—straight/curved, number of sides/corners; regular vs. irregular) and begin distinguishing 2D vs. 3D objects in daily life (balls vs. blocks). They sequence daily events (morning routine, snack, outside time, bedtime) to build time concepts and begin telling time to the hour on analog clocks. Money play introduces pennies, nickels, and dimes—counting combinations in play stores or 'bank' games. Logical reasoning grows through 'What doesn't belong?' activities and simple attribute logic puzzles. Real-life connections make math practical: following schedules, paying for small items, building with blocks. Teachers use ten-frames, part-part-whole mats, shape sorting mats, and daily calendar routines. This level bridges early number sense to place-value readiness while embedding geometry properties, time/money life skills, and logical classification—creating a richer, more connected experience than isolated counting drills.
Level 4	**Counting/Comparing/Ordering to 20+, Pattern Extension & Early Data/Chance**	Children count aloud to 20+ with accuracy and one-to-one correspondence, compare and order numbers up to 20, and begin seeing structure in the number sequence. Patterns extend to AAB, ABC, and simple growing patterns with objects, shapes, and numbers—foundational algebraic thinking. Data and logic integrate early: children collect simple data (favorite color, pet, lunch choice) via class surveys, sort into categories, and represent with concrete picture graphs or tallies. Probability/chance language begins intuitively through experiments: 'Will this spinner land on red or blue? Can it land on both? What is certain/impossible?' Coin flips or color spinners introduce 'equally likely,' 'more likely,' 'less likely.' Geometry deepens with composing/decomposing shapes using pattern blocks and tangrams; symmetry hunts in flags, letters, and nature. Real-life: counting larger sets (books, blocks), ordering events or objects by size, simple surveys ('How many like apples vs. bananas?'). Teachers model data collection and 'I wonder...' questions. This level deliberately balances extended counting with early data, probability intuition, advanced patterns, and geometry composition—avoiding the original's narrow counting focus and preparing logical/data minds alongside number skills.
Level 5	**Early Place Value (Tens & Ones), Concrete +/- Strategies & Measurement Foundations**	Children group objects into tens and ones, developing true place-value understanding: '2 tens and 7 ones is the same as 27.' They skip-count by 2s, 5s, and 10s from various starting points and use this to count larger collections efficiently. Addition and subtraction move from purely concrete to strategies (counting on, counting back, making ten, doubles, near-doubles) modeled first with objects and drawings, then connected to symbols. Measurement begins with non-standard units (paper clips, cubes) then transitions to standard inches/centimeters for length; capacity and weight explored with water and balances. Geometry continues: children measure perimeter informally with string or tiles and explore area as 'how many squares cover the shape.' Time-telling advances to hour and half-hour; money combinations expand. Logic/problem-solving emphasis: students explain strategy choice ('I used making ten because 8+2 is easy') and check reasonableness. Real-life projects: measuring classroom objects, building with exact counts of tens, simple recipes. Teachers use base-10 blocks, ten-frames, number lines, and think-alouds. This level cements place value as the bridge to all future operations while integrating measurement, early +/- strategies, and continued geometry—creating a solid conceptual base before written algorithms or multiplication.
Level 6	**Numbers to 100, Expanded Form, Skip Counting & Shape Classification + Symmetry**	Children read, write, compare, and order numbers to 100, seeing them as organized systems rather than digit strings. They represent numbers in expanded form (47 = 40 + 7) and with base-10 materials, deepening place-value fluency. Skip-counting by 5s and 10s (and 2s) from any number builds multiplication readiness. Addition and subtraction strategies within 20 become fluent; students solve word problems and explain inverse relationship (addition undoes subtraction). Geometry advances: classify 2D shapes by number of sides/angles and presence of right angles or symmetry; explore congruence and simple transformations (slides, flips, turns) with pattern blocks. Data interpretation strengthens: students read and ask questions about picture graphs and bar graphs. Real-life: hundreds charts for games, measuring longer distances, money to $1.00, shape hunts for symmetry in architecture/nature. Teachers introduce 'prove it' language and multiple representations. This level solidifies two-digit number sense and operations while expanding geometry classification and data skills—preparing for regrouping and multi-digit work without rushing algorithms before understanding.
Level 7	**Written 2-Digit +/- (No Regrouping), Word Problems, Data Graphs & Logical Reasoning**	Children add and subtract two-digit numbers without regrouping, aligning by place value and using drawings or partial algorithms while always explaining with place-value language. Multi-step word problems appear, requiring students to break problems into parts and choose strategies. Data skills deepen: students design simple surveys, collect data, create tally charts and bar graphs, and interpret results to make decisions. Logical reasoning and problem-solving are explicit: students justify solutions ('I added the tens first, then the ones'), consider alternative strategies, and check reasonableness ('Does 45 + 32 = 77 make sense? It's a bit more than 70, yes'). Geometry connects to measurement: students measure perimeter of polygons with rulers or string and begin area as covering with square units (tiles or grid paper). Time and money applications expand to real scenarios (schedules, classroom store). Teachers model 'think-aloud' problem solving and require students to explain thinking in words, drawings, and numbers. This level introduces written computation meaningfully (after deep place-value work) while building data literacy, logical justification, and early measurement formulas—filling gaps in original by making reasoning and data central rather than add-ons.

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Level 8	**Regrouping (Carrying & Borrowing) with Place Value, Multi-Step Problems & Probability Intro**	Children learn regrouping (often called 'carrying' in addition and 'borrowing' in subtraction) as a logical extension of place value: when adding ones and reaching 10 or more, compose a new ten and carry/record the 1 in the tens place. Subtraction borrowing is taught as decomposing a ten into 10 ones when there aren't enough ones. This is always introduced FIRST with base-10 blocks or drawings so students understand WHY before learning the standard algorithm as an efficient recording method. Multi-step word problems increase in complexity. Probability deepens: students conduct repeated experiments (coin flips, dice rolls, spinners), record outcomes in tallies or tables, describe results ('Heads came up about half the time'), and use chance language (equally likely, more/less likely, impossible/certain) to predict. Geometry: students explore angles informally (right, acute, obtuse) through shape corners and paper folding; classify quadrilaterals by parallel sides and right angles. Data: students compare graphs and draw conclusions. Teachers emphasize 'Does this make sense?' and model explaining the 'why' of regrouping. This level explicitly demystifies regrouping (a common source of confusion) with conceptual foundation first, while integrating probability experiments and deeper geometry classification—addressing original gaps in explanation and stats/prob timing.
Level 9	**Fluent +/- within 100 (with/without Regroup), Estimation, Perimeter/Area & Data Interpretation**	Children achieve fluency with addition and subtraction within 100, choosing efficient strategies (mental math, drawings, algorithm) and always estimating first for reasonableness. Multi-step problems integrate money and measurement contexts. Geometry measurement deepens: students calculate perimeter of polygons and rectangles using addition or formulas; area is explored as multiplication of side lengths (arrays) and by counting square units. Data skills advance: students interpret bar graphs, picture graphs, and simple line plots; ask/answer comparison and 'how many more/less' questions; begin to notice patterns or outliers in class data. Logical problem-solving: students articulate multiple solution paths, critique others' reasoning respectfully, and use inverse operations to check. Real-life projects: plan a small classroom event budget, measure and fence a garden plot (perimeter), or survey and graph preferences. Teachers continue think-alouds and require clear written/oral explanations. This level consolidates two-digit operations with deep conceptual understanding (no rote algorithms), strengthens measurement formulas, and makes data interpretation a core skill—preparing for multi-digit work and stats in later levels while filling the original's rushed pacing gap.
Level 10	**Hundreds Place Value, Multiplication/Division Concepts as Groups & Arrays, 3D Shapes & Growing Patterns**	Children develop hundreds as a place-value unit, reading/writing/comparing/composing numbers to 999 with base-10 materials and expanded form. They see the structure of our number system (ones, tens, hundreds) and round to nearest 10 or 100. Multiplication and division concepts are introduced conceptually: multiplication as repeated addition and equal groups/rectangular arrays; division as sharing equally or forming equal groups (how many in each group or how many groups). Skip-counting connects directly to multiplication facts (2s, 5s, 10s). Geometry expands to 3D shapes: identify faces, edges, vertices of cubes, rectangular prisms, cylinders, cones, spheres; sort and compare 2D vs. 3D. Measurement: time to nearest 5 minutes and simple elapsed time; liquid volume (cups, pints, quarts, liters) and mass/weight with balances. Patterns: numeric growing patterns and 'rule' finding (e.g., add 3 each time). Problem-solving: students model * and / stories with objects/drawings and explain the meaning. Real-life: hundreds counting in collections, array hunts in gardens or buildings, 3D shape scavenger hunts, cooking measurements. This level provides strong conceptual introduction to multiplication/division (preventing rote memorization without understanding) while expanding place value, 3D geometry, and measurement—addressing the original's fast jump into mult without foundations.
Level 11	**Multiplication Facts Strategies & Fluency, Division as Inverse, Triangle/Quad Classification & Logical Justification**	Children build multiplication fact fluency (0–10) using strategies: arrays, skip-counting, fact families, distributive property (breaking apart), and doubles/near-doubles. Division is explicitly taught as the inverse of multiplication (fact families) and through sharing/ repeated-subtraction stories. Word problems for both operations increase in complexity and require modeling, solving, and explaining. Geometry deepens classification: triangles by sides (equilateral, isosceles, scalene) and angles (acute, right, obtuse); quadrilaterals by parallel sides, right angles, and equal sides—beginning hierarchy understanding (all squares are rectangles, but not all rectangles are squares). Symmetry and congruence are applied to classify and create designs. Logical problem-solving is central: students justify solutions with models or equations, consider multiple paths, and prove answers. Data: students create and interpret line plots from measurement data (e.g., classmates' heights or pencil lengths). Real-life: array-based games, fact fluency apps/games, shape classification hunts, measuring for line plots. Teachers require 'prove it with a model or drawing' and celebrate efficient strategies. This level ensures multiplication/division facts are understood conceptually (not just memorized) while advancing geometry classification and logical justification—filling the original's rushed mult intro and weak geometry progression.
Level 12	**Multi-Digit Multiplication Strategies, Long Division Intro, Fraction Foundations & Line Plots/Probability Experiments**	Children apply strategies (partial products, area models) to multiply multi-digit numbers (2- or 3-digit × 1-digit), always estimating first and explaining with place-value reasoning. Long division is introduced conceptually: dividing multi-digit dividends by single-digit divisors using repeated subtraction or grouping, recording steps, stating quotient and remainder, and verifying by multiplying back. Remainders are interpreted in context ('leftover' or 'how many full groups + extra'). Fraction foundations are built visually: partition wholes into equal parts (paper folding, fraction circles, bars, sets); name unit and non-unit fractions (1/2, 1/3, 2/3, 3/4); compare fractions with same denominator or same numerator using models. Data/probability: create line plots from repeated measurement; conduct probability experiments, record frequencies, express simple probability as fraction of outcomes (e.g., '3 out of 8 spins landed on red'). Geometry: deepen symmetry and begin angle measurement with protractors in context of shapes. Real-life: partial-products for pricing multiple items, division for sharing fairly or grouping, fractions in cooking/recipes, probability games. Teachers model strategies with base-10 or area models and require students to explain each step. This level provides conceptual depth for algorithms (preventing 'I don't get why we do this') and integrates fractions early with visuals—addressing original gaps in fraction timing and probability
Level 13	**Multi-Digit Mult (2×2), Division with Remainders in Context, Equivalent Fractions & Volume as Filling**	Children extend multi-digit multiplication to 2-digit × 2-digit using area models or partial products, estimating and justifying with place value. Division deepens: larger dividends, remainders interpreted meaningfully in word problems (e.g., 'How many full teams of 4 can we make from 27 students? How many left over?'). Equivalent fractions are generated and justified with models (multiplying numerator and denominator by same number keeps value same); fractions compared and ordered using benchmarks (0, 1/2, 1) or common denominators visually. Volume introduced as 'how many unit cubes fill a rectangular prism'—students build prisms with cubes, count, and connect to dimensions (length × width × height preview). Geometry hierarchy deepens: classify 2D shapes with full justification; explore additive angle measure (adjacent angles). Data: analyze graphs for patterns, begin simple measures of center (mode, median) with concrete data sets. Problem-solving: multi-step problems integrating operations, fractions, and measurement. Real-life: scaling recipes (fractions), packing boxes (volume), team grouping (division with remainders), probability in games/sports. Teachers use concrete volume models and require students to explain equivalence with visuals. This level strengthens multi-digit algorithms with understanding, advances fractions to equivalence, and introduces volume conceptually—preparing for formulas and deeper stats while maintaining balance
Level 14	**Fraction +/- (Like Denominators), Decimal Place Value & Money, Coordinate Graphing & Order of Operations Intro**	Children add and subtract fractions with like denominators using models (fraction bars, circles) then connect to symbolic work; results are simplified when appropriate. Decimals are introduced as an extension of place value and fractions: read, write, compare, and operate with decimals to hundredths in money and measurement contexts (e.g., $3.45, 2.75 meters). Coordinate graphing begins in the first quadrant: plot and read points on a grid (treasure-map style), connect to real maps or game boards. Order of operations introduced lightly with parentheses (PEMDAS foundation): students evaluate simple expressions and explain why order matters ('Do what's inside the parentheses first'). Geometry measurement: area of rectangles with formula, perimeter of composite shapes. Volume formula (V = l × w × h) practiced with cubes and then symbolically. Problem-solving integrates fractions, decimals, graphing, and geometry in multi-step tasks. Real-life: money calculations with decimals, map reading, simple game design on coordinate grid, following order in recipes or instructions. Teachers model with money manipulatives and coordinate grids. This level bridges fractions to decimals (critical connection often missing), introduces coordinate plane and order of operations naturally, and consolidates volume—creating a smooth ramp to 5th-grade capstone without overwhelming any single level.

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Level	Summary	Description
Level 15	**Fraction Ops (Unlike Denoms & × Whole), Decimal Operations, Shape Hierarchies & Angle Measure, Data Analysis (Mean/Median/Mode)**	Children add and subtract fractions with unlike denominators by finding common denominators or using models; multiply a fraction by a whole number (e.g., 3 × 1/4 = 3/4) with visual support and real contexts (recipes, scaling). Decimal operations (+ and – to hundredths) become fluent in money, measurement, and data contexts. Geometry: full hierarchical classification of 2D shapes with justifications; additive angle measure (adjacent angles sum); classify 3D shapes by attributes and nets. Data analysis deepens: calculate and interpret mean (average as 'balancing point' or fair share), median, mode, and range for appropriate data sets; identify outliers and discuss impact. Probability: predict outcomes from experimental data and compare to theoretical (simple cases). Multi-step problem-solving integrates all prior skills with clear communication of reasoning. Real-life projects: scale a recipe (fractions), budget a party with decimals, design a garden (area, perimeter, volume), analyze class survey data with measures of center. Teachers emphasize justification and multiple representations. This level advances fraction and decimal operations meaningfully, deepens geometry hierarchy and angle concepts, and makes data analysis (mean/median/mode) accessible—preparing students for capstone integration and middle-school readiness while maintaining strand balance.
Level 16	**Integrated Fluency (Whole Number, Fraction, Decimal Ops), Algebraic Thinking Intro & Composite Shapes/Volume**	Children demonstrate fluent, flexible computation across whole numbers, fractions, and decimals—choosing efficient strategies and explaining why. Algebraic thinking emerges: write and evaluate simple expressions with variables (e.g., n + 5 when n=3), solve one-step equations in context, and describe patterns with rules. Geometry measurement mastery: calculate area and perimeter of composite (irregular) shapes by decomposing into rectangles; volume of rectangular prisms with formula and in real packing/design problems; surface area preview with nets. Problem-solving becomes highly strategic: students select from multiple strategies, use estimation fluently, break complex problems into manageable parts, and communicate reasoning clearly in words, drawings, and equations. Data/probability: design simple investigations, collect/organize/represent/analyze data in various graphs, draw conclusions, and conduct probability experiments comparing experimental vs. theoretical results. Real-life capstone-style projects: plan a garden or room redesign (area, perimeter, volume, cost with decimals), analyze sports or weather data, create probability games. Teachers facilitate student-led explanations and peer feedback. This level synthesizes prior learning into flexible, integrated performance—filling any remaining gaps and building confidence for the final capstone levels.
Level 17	**Advanced Multi-Domain Problem Solving, Fraction/Decimal Flexibility, Full Geometry Justification & Probability Predictions**	Children tackle advanced, novel multi-step problems that integrate number operations (whole, fraction, decimal), geometry (measurement + classification), data analysis, and probability—requiring strategic planning, tool selection, and clear justification. Fraction and decimal work emphasizes equivalence, mental strategies, and flexible computation (e.g., 0.75 = 3/4). Geometry: complete hierarchical classification with proofs; solve problems involving area, perimeter, volume, surface area (nets), and angle relationships. Probability: make predictions based on experimental data and theoretical models; calculate simple probabilities as fractions and use to inform decisions. Logical reasoning deepens: error analysis (find and fix mistakes, explain what went wrong), multiple solution paths, proof by example or counterexample, and deductive thinking in puzzles. Real-life projects: design a budget and layout for a small event or business (decimals, area, volume, graphs), analyze game fairness (probability), or create instructional math games for younger students. Teachers act as facilitators, prompting deep explanation and reflection. This level challenges students to apply everything flexibly and justify rigorously—preparing them for the independent capstone while addressing any lingering gaps in integration or advanced reasoning.
Level 18	**Comprehensive Review & Fluency, Complex Integration, Full Data Cycle & Deductive Logic**	Children engage in comprehensive review and targeted fluency building across all domains, identifying personal strengths and areas for growth. Complex multi-step problems and projects require full integration: e.g., design a school garden or playground (area/perimeter/volume calculations, material costs with decimals, scaled drawings on coordinate grid, data collection on plant growth or usage, probability of weather impacting plans). Full data cycle practiced: pose statistical question, design collection method, organize/represent (multiple graph types), analyze (measures of center, variability, outliers), interpret, and communicate conclusions. Deductive logic and puzzles strengthen reasoning (e.g., attribute logic, 'what's my rule?', proof puzzles). Geometry and measurement fully mastered: all classifications justified, all formulas applied flexibly to composite figures and real designs. Teachers facilitate student choice in projects and emphasize reflection, self-assessment, and peer teaching. This level ensures no gaps remain before capstone and builds agency, communication, and deep understanding—students see math as a connected, powerful tool for real projects rather than isolated skills.
Level 19	**Capstone Preparation: Flexible Thinking, Strategy Selection, Justification & Communication**	Children prepare for the final capstone by engaging in challenging, open-ended tasks that demand flexible thinking, strategic strategy selection, deep justification, and clear communication. Tasks include error analysis of complex problems, designing multiple solution paths, creating math games or instructional materials for younger students, and tackling novel real-world scenarios (e.g., plan a community event with budget, layout, scheduling, data tracking, and probability of attendance). Students practice explaining their thinking to different audiences (peers, younger children, adults) and receiving/using feedback. Geometry, fractions, decimals, volume, graphing, and probability are reviewed through application rather than isolated drills. Teachers shift to coaching/facilitation: prompt deeper justification, celebrate efficient and creative strategies, and help students articulate connections ('How does what you learned about fractions help with decimals here?'). Self-assessment and goal-setting are explicit. This level builds the confidence, communication skills, and integrated mastery needed for independent capstone performance—addressing the original's abrupt jump to capstone without sufficient synthesis practice.
Level 20	**5th Grade Capstone Integration: Synthesis, Fluency, Reasoning & Real-Life Application**	Children synthesize all prior skills into fluent, integrated, independent performance. No new concepts are introduced. Emphasis is on multi-step problem solving, strategic reasoning, estimation, clear explanations, flexible application across contexts, and reflection. Students complete a comprehensive set of 10–15 varied tasks (or a major culminating project) requiring: fluent whole-number and decimal operations, fraction operations (all types), geometric classification and measurement (area, perimeter, volume, angles), coordinate graphing, data analysis (graphs, measures of center, outliers), probability reasoning, order of operations/expressions, and logical justification. Tasks include complex real-world scenarios (e.g., design and budget a small business or community project, analyze sports or science data, create probability-based games). Students demonstrate fluency, choose efficient strategies, show clear organized work, explain reasoning across domains, check reasonableness, and reflect on their mathematical growth. This capstone confirms readiness for middle school math with strong conceptual understanding, procedural fluency, and the ability to communicate and apply math powerfully in novel situations—fulfilling the original vision while ensuring the path was comprehensive, logical, and gap-free.